Remarks on A Formulae-as-Types Notion of Control

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Dedicated to P-L Curien on the occasion of his 60th birthday.

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THE FORMULAE-AS-TYPES NOTION OF CONSTRUCTION

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Dedicated to H. B. Curry on the occasion of his 80th birthday.

The following consists of notes which were privately circulated in 1969. Since they have been referred to a few times in the literature, it seems worth while to publish them. They have been rearranged for easier reading, and some inessential corrections have been made.
Control Calculi of Matthias Felleisen

\[(\lambda x. M) V \xrightarrow{\beta_v} M[V/x]\]  \hspace{2cm} (\beta_v)

\[(A M) N \xrightarrow{A_L} A M\]  \hspace{2cm} (A_L)

\[V(A N) \xrightarrow{A_R} A N\]  \hspace{2cm} (A_R)

\[(C M) N \xrightarrow{C_L} C \lambda k. M(\lambda f. k(f N))\]  \hspace{2cm} (C_L)

\[V(C N) \xrightarrow{C_R} C \lambda k. N(\lambda v. k(V v))\]  \hspace{2cm} (C_R)
A Formulae-as-Types Notion of Control

- Type $C$ as double negation elimination, $\neg\neg\alpha \rightarrow \alpha$
- Type Scheme’s call/cc as Peirce’s law, $(\alpha \rightarrow \beta) \rightarrow \alpha \rightarrow \alpha$
- CPS as an embedding of Classical into Intuitionistic logic
- Computation with excluded middle as a kind of restricted backtrack
- Recover expected computation from classical definitions such as $\alpha \land \beta \equiv \neg(\neg\alpha \rightarrow \neg\beta)$ and $\alpha \lor \beta \equiv \neg\neg\alpha \rightarrow \beta$

Limitations: Natural deduction approach, call-by-value only.
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- Texas has the death penalty, so murder was too risky
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- I was driving Matthias Felleisen crazy by dogmatically evangelizing types.
- Matthias’ anti-types tirades made me very very angry.
- Texas has the death penalty, so murder was too risky.
- Sweet revenge — type Matthias’ λC language!
My inspiration: Natural Deduction, Dag Prawitz, 1965

\[
\begin{align*}
\frac{\neg F}{\Sigma} \\
\frac{\lambda}{\Pi_1}
\end{align*}
\]

\(\{\neg F\}\) is the set of assumptions discharged by \(\alpha\). \(F\) has one of the shapes \(B \& C\), \(B \supset C\), and \(\forall xB\). We remove this application of the \(\xi\)-rule by transforming \(\Pi\) in the respective cases to:

\[
\begin{align*}
\frac{\Sigma}{\lambda} \\
\frac{\Sigma}{\lambda} \\
\frac{\Sigma}{\lambda}
\end{align*}
\]

\[
\begin{align*}
\frac{B \& C}{B} & \quad \frac{B \& C}{C} & \quad \frac{B \supset C}{C} & \quad \frac{\forall xA}{A^*_\alpha} \\
\frac{B}{\neg B} & \quad \frac{\neg C}{C} & \quad \frac{\neg C}{C} & \quad \frac{\neg \forall xA}{\neg A^*_\alpha}
\end{align*}
\]

\[
\begin{align*}
\frac{\Sigma}{\lambda} \\
\frac{\Sigma}{\lambda} \\
\frac{\Sigma}{\lambda}
\end{align*}
\]

\[
\begin{align*}
\frac{\Sigma}{\lambda} \\
\frac{\Sigma}{\lambda}
\end{align*}
\]
Write reduction from Prawitz as

\[ C(\lambda x \neg(\alpha \rightarrow \beta) \cdot N \perp) \alpha \rightarrow \beta \rightarrow \lambda a \cdot \alpha \cdot C(\lambda k \neg \beta \cdot N[(\lambda f \alpha \rightarrow \beta \cdot k(f \cdot a))/x]) \]
Write reduction from Prawitz as

\[ \mathcal{C}(\lambda x \neg (\alpha \rightarrow \beta) \cdot N \bot) \alpha \rightarrow \beta \rightarrow \lambda a \alpha . \mathcal{C}(\lambda k \neg \beta . N[(\lambda f \alpha \rightarrow \beta . k(f\ a))/x]) \]

Fiddle with this a bit and write

\[ \mathcal{C}(M \neg \neg (\alpha \rightarrow \beta)) \alpha \rightarrow \beta \rightarrow \lambda a \alpha . \mathcal{C}(\lambda k \neg \beta . M(\lambda f \alpha \rightarrow \beta . k(f\ a)))) \]
Write reduction from Prawitz as

\[ C(\lambda x^{\neg(\alpha \to \beta)}.N^{\perp})^{\alpha \to \beta} \longrightarrow \lambda a^{\alpha}.C(\lambda k^{\neg \beta}.N[(\lambda f^{\alpha \to \beta}.k(f\ a))/x]) \]

Fiddle with this a bit and write

\[ C(M^{\neg(\alpha \to \beta)})^{\alpha \to \beta} \longrightarrow \lambda a^{\alpha}.C(\lambda k^{\neg \beta}.M(\lambda f^{\alpha \to \beta}.k(f\ a)))) \]

Then plug into an application context to derive a typed \( C_L \)

\[ C(M^{\neg(\alpha \to \beta)})^{\alpha \to \beta} N^{\alpha} \longrightarrow C(\lambda k^{\neg \beta}.M(\lambda f^{\alpha \to \beta}.k(f\ N)))^{\beta} \]
The two $C$ rules in Natural Deduction style

\[
\begin{align*}
\Sigma_1 \\
\neg(A \rightarrow B) \\
A \rightarrow B \\
\hline
\Sigma_2 \\
A
\end{align*}
\] \quad \Rightarrow \quad 
\begin{align*}
\Sigma_1 \\
\neg(A \rightarrow B) \\
\hline
\Sigma_2 \\
\neg(B) \\
\hline
B
\end{align*}

\[
\begin{align*}
\Sigma \\
\neg\neg A \\
A \rightarrow B \\
\hline
\Sigma \\
A \\
\neg\neg A \\
\hline
\neg\neg B \\
\hline
B
\end{align*}
\]
On a Formal Correspondence Between 
\(\lambda\text{-}C\)-Terms and Classical Proofs

— Extended Abstract —

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October 28, 1988
December 20, 1988

Dear Author:

I am sorry to have to tell you that your paper

*On a Formal Correspondence between A-C-Terms and Classical Formulas*

was not among those selected by the program committee for presentation in June. Far more good papers were submitted than the program could possibly accommodate and we were forced to reject papers that were clearly publishable in a journal.

Thank you anyhow for submitting your paper to LICS 89 and I hope that you will attend the meeting in June.

Yours sincerely,

Rohit Parikh
Program chair