Some Varieties of Constructive Finiteness

Erik Parmann

University of Bergen

Presented at:
TYPES 2014

May 12, 2014

Thierry Coquand and Arnaud Spiwack.
Constructively finite?
In *Contribuciones científicas en honor de Mirian Andrés Gómez*, pages 217–230. Universidad de La Rioja, 2010.

Marc Bezem, Keiko Nakata, and Tarmo Uustalu.
On streams that are finitely red.
Logical Methods in Computer Science, 8(4), 2012.

Enumerated (Kuratowski finite)

↓

Bounded Size

↓

Noetherian

↓

Streamless

Enumerated (Kuratowski finite)

↓

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↓

Streamless

Sum and Product

Enumerated (Kuratowski finite)	Sum and Product
$\downarrow$	
Bounded Size	Sum and Product
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Enumerated (Kuratowski finite)	Sum and Product
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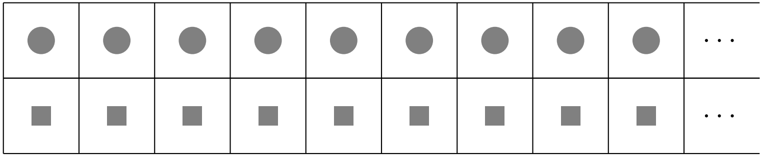
Streamless	Sum and Product (decidable eq)

Streamless is closed under product (given decidable equality):



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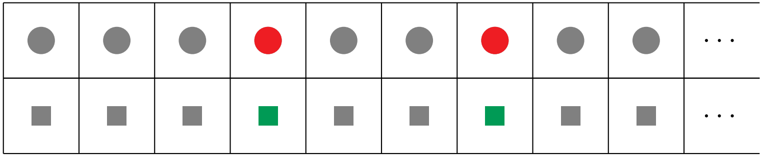
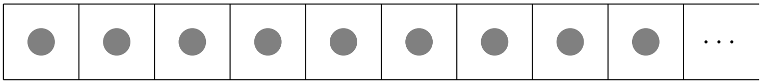


Producing n equal elements:



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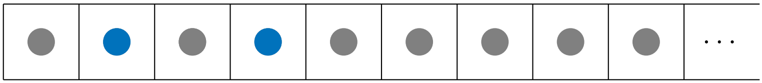

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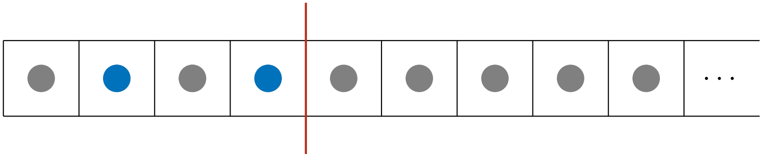

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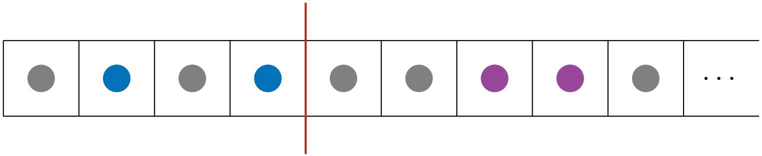

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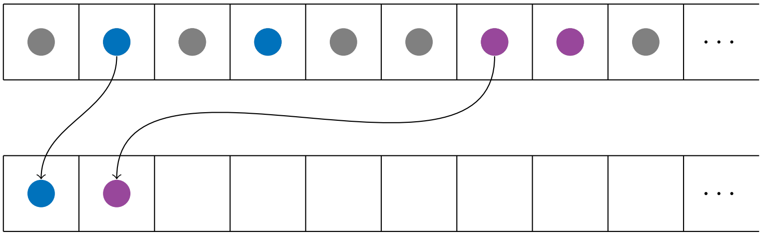

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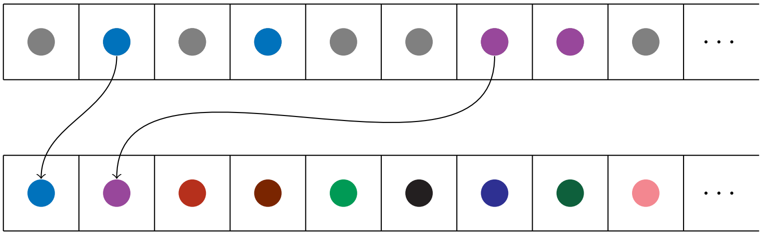

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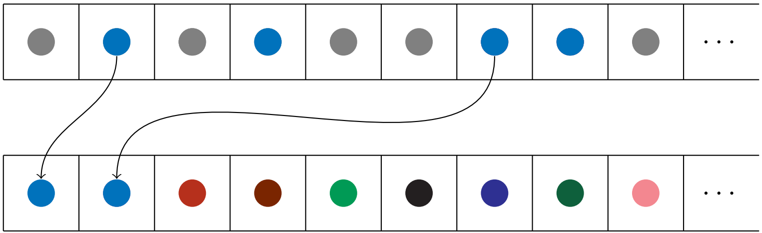

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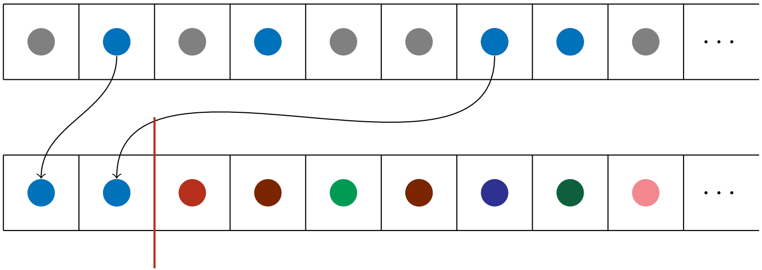

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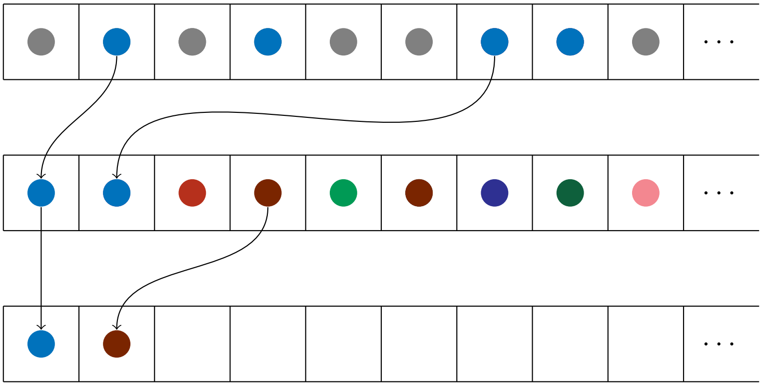

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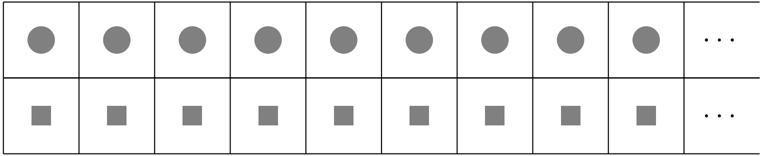

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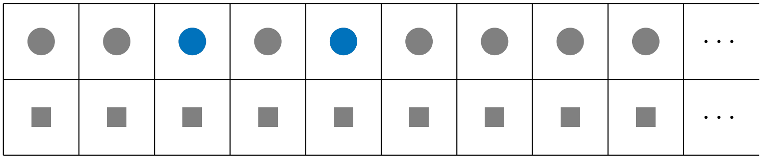

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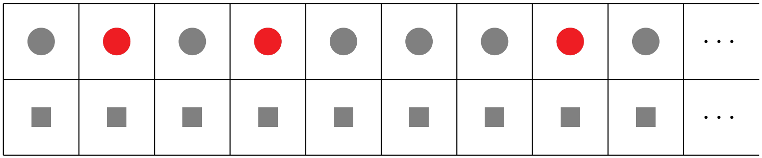

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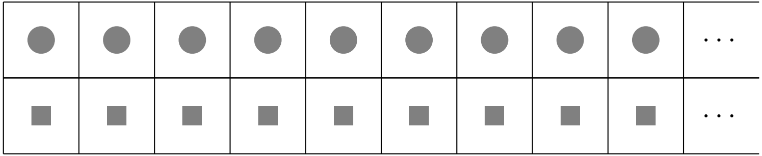

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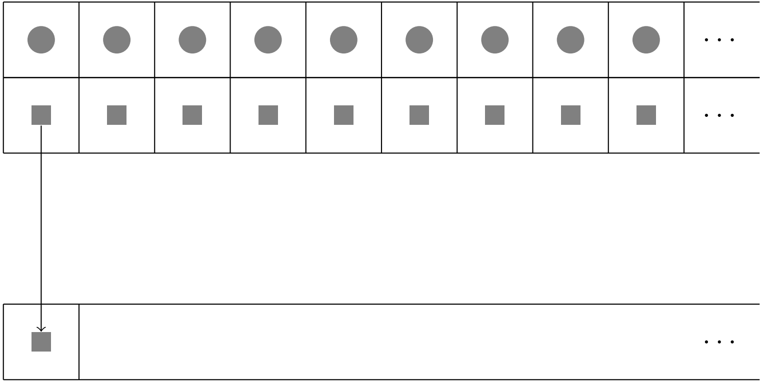

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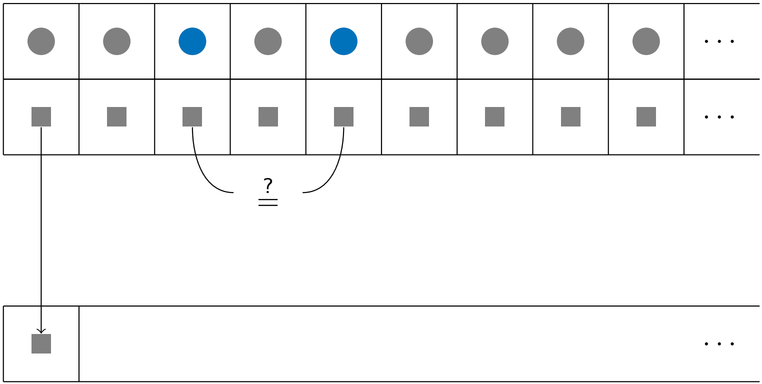

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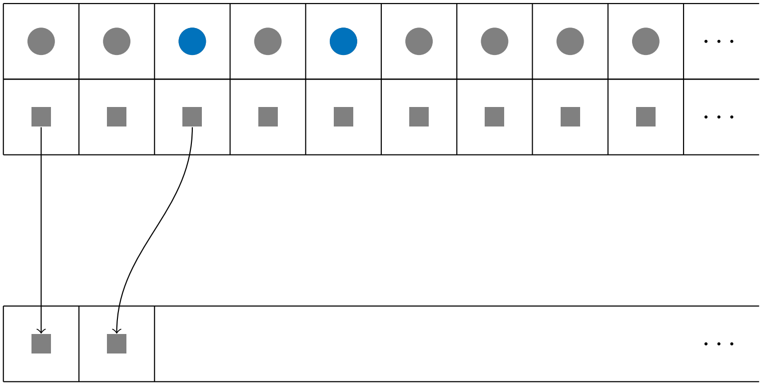

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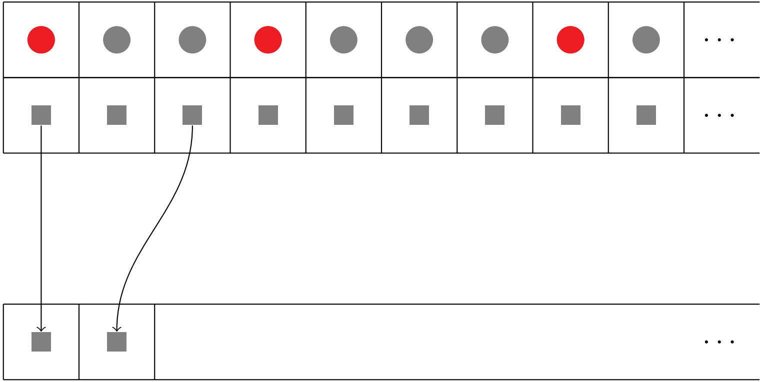

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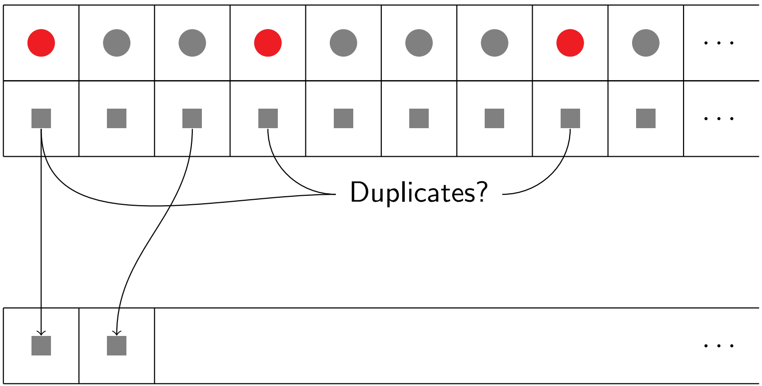

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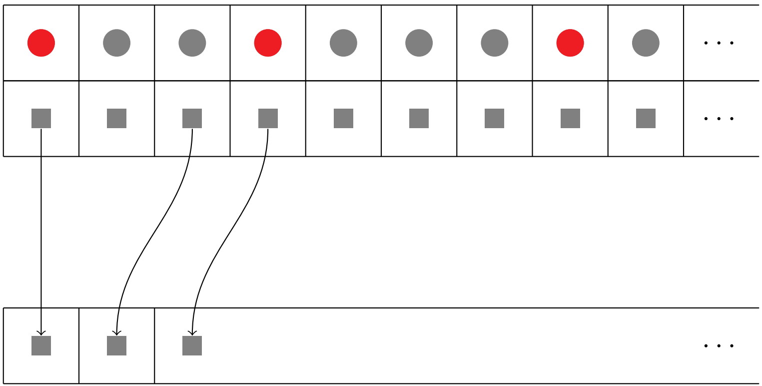

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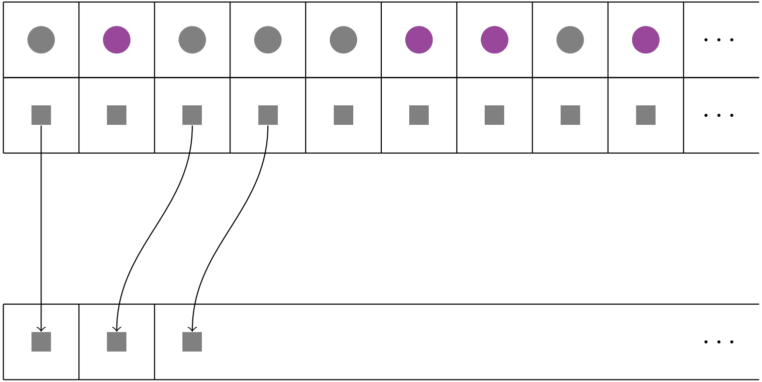

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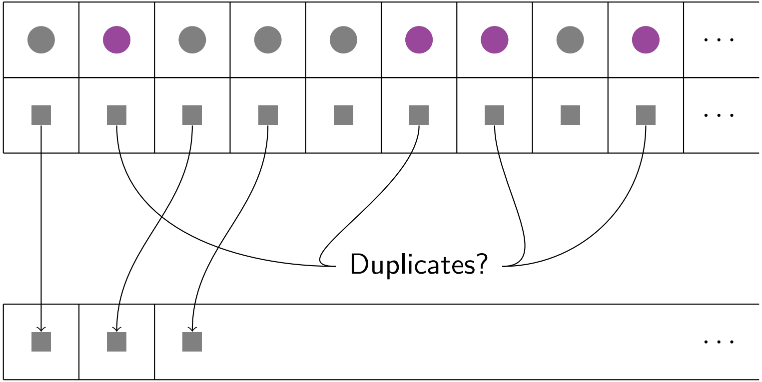

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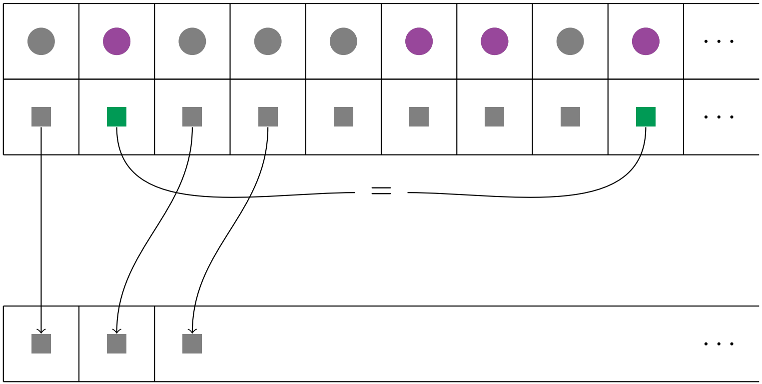

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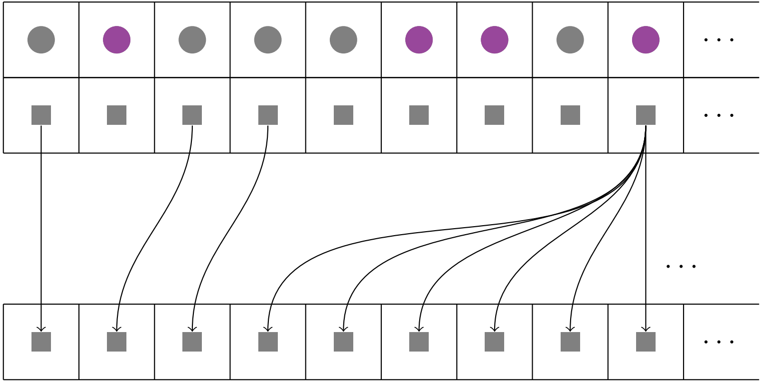

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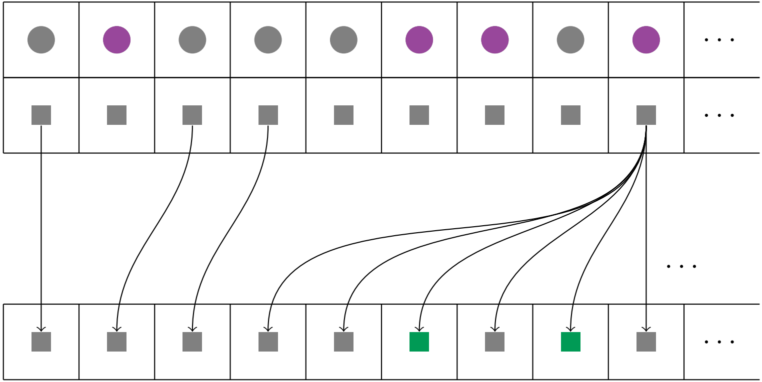

Streamless and function extensionality implies decidable equality:

$$1 \stackrel{?}{=} 2$$

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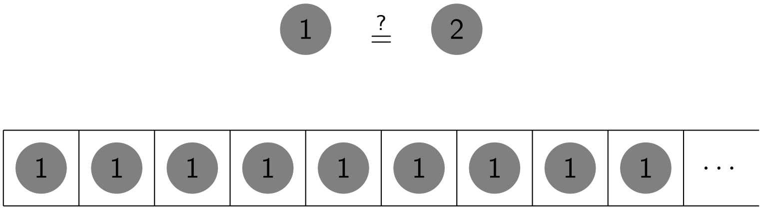

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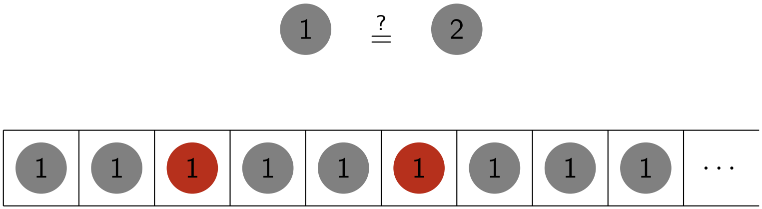

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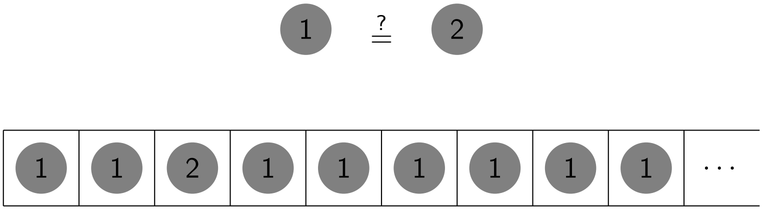

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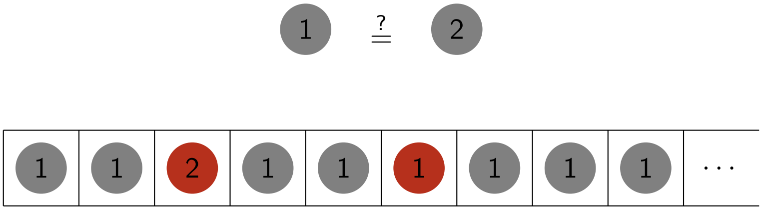

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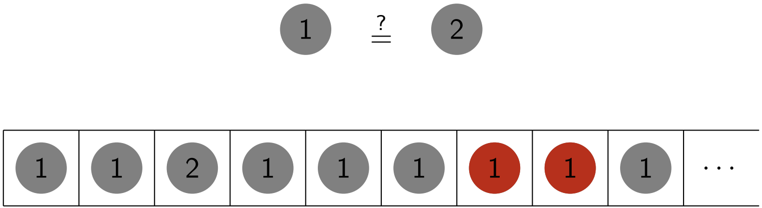

TODO:

- Can it work withouth decidable equality?
  - With Markov's Principle?
  - If one of the sets are Noetherian?

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- Can it work withouth decidable equality?
  - With Markov's Principle?
  - If one of the sets are Noetherian?
- Look into natural definitions of finiteness which does not give decidable equality with function extensionality.

Thanks,

http://folk.uib.no/epa095/

Definition (Markov's Principle, MP)

For any decidable predicate:

$$\neg\neg\exists n : \mathbb{N}, P(n) \to \exists n : \mathbb{N}, P(n).$$

Definition (Limited Principle of Omniscience (LPO))

For any decidable predicate P, we have

$$(\forall n : \mathbb{N}, P(n)) \vee (\exists n : \mathbb{N}, \neg P(n)).$$

Definition (Weak Limited Principle of Omniscience (WLPO))

For any decidable predicate P, we have

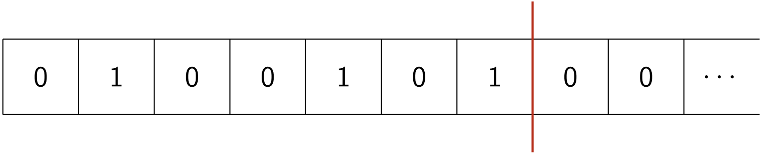
$$(\forall n : \mathbb{N}, P(n)) \vee (\neg \forall n : \mathbb{N}, P(n)).$$

Fact

$$(MP \wedge WLPO) \iff LPO$$

Definition (Eventually always false (Eaf))

$$\exists n : \mathbb{N}, \forall m : \mathbb{N}, m \geq n \to f(m) = 0.$$



Definition (Bounded(f))

$$\exists n : \mathbb{N}, \forall k : \mathbb{N}, \texttt{NrOf1}_f\ k \leq n.$$

E.g with $n = 5$:

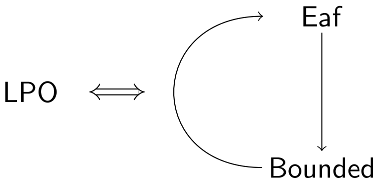
0	1	0	0	1	0	1	0	0	$\cdots$

Definition (Sb)

$$\exists n : \mathbb{N}, (\forall k : \mathbb{N}, \texttt{NrOf1}_f\ k \leq n \land \neg \forall k : \mathbb{N}, \texttt{NrOf1}_f\ k \leq n - 1)$$

E.g with $n = 5$:

0	1	0	0	1	0	1	0	0	$\cdots$



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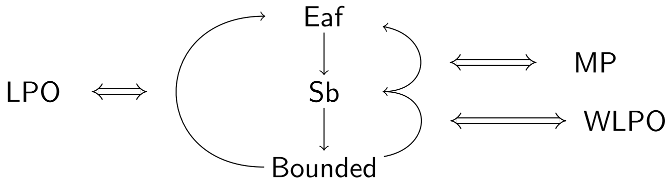
Definition (Limited Principle of Omniscience, LPO)

$$(\forall n : \mathbb{N}, P(n)) \vee (\exists n : \mathbb{N}, \neg P(n)).$$



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Eaf

LPO $\iff$

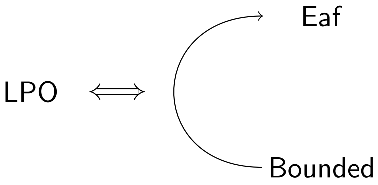
Bounded

$$\exists n : \mathbb{N}, \forall m : \mathbb{N}, m \geq n \to f(m) = 0.$$

$$\exists n : \mathbb{N}, \forall k : \mathbb{N}, \texttt{NrOf1}_f\ k \leq n.$$

Definition (Markov's Principle, MP)

$$\neg\neg\exists n : \mathbb{N}, P(n) \to \exists n : \mathbb{N}, P(n).$$



$$\exists n : \mathbb{N}, \forall m : \mathbb{N}, m \geq n \to f(m) = 0.$$

$$\exists n : \mathbb{N}, \forall k : \mathbb{N}, \texttt{NrOf1}_f \; k \leq n.$$

Definition (Weak Limited Principle of Omniscience (WLPO))

$$(\forall n : \mathbb{N}, P(n)) \vee (\neg \forall n : \mathbb{N}, P(n)).$$