Modular and lightweight certification of polyhedral abstract domains

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Verimag, Grenoble

May 14, 2014

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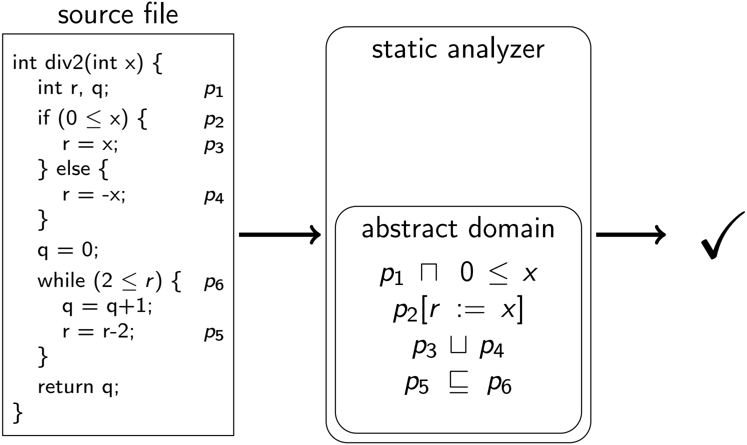

source file
int div2(int x) {
int r, q; p_1
if (0 ≤ x) { p_2
r = x; p_3
} else {
r = -x; p_4
}
q = 0;
while (2 ≤ r) { p_6
q = q+1;
r = r-2; p_5
}
return q;
}
static analyzer
abstract domain
$p_1 \sqcap 0 \leq x$
$p_2[r := x]$
$p_3 \sqcup p_4$
$p_5 \sqsubseteq p_6$
✓

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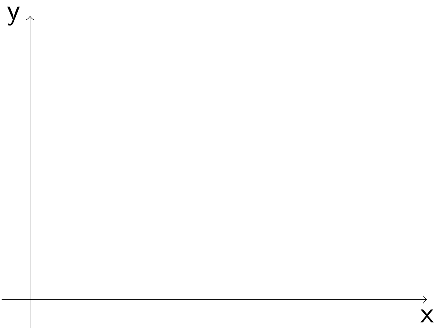
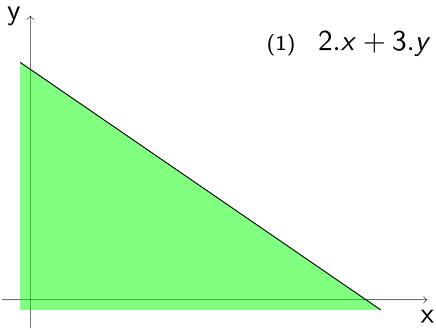
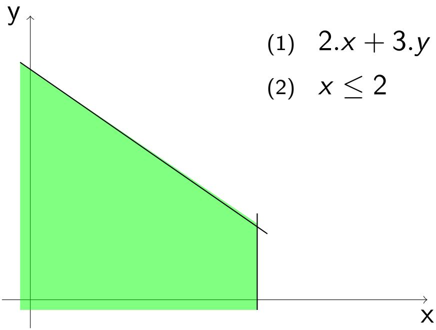

y
x

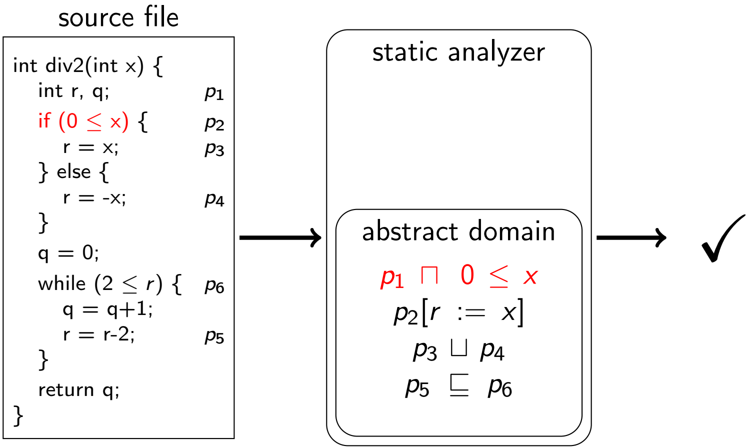


(1) $2.x + 3.y \leq 6$



(1) $2.x + 3.y \leq 6$

(2) $x \leq 2$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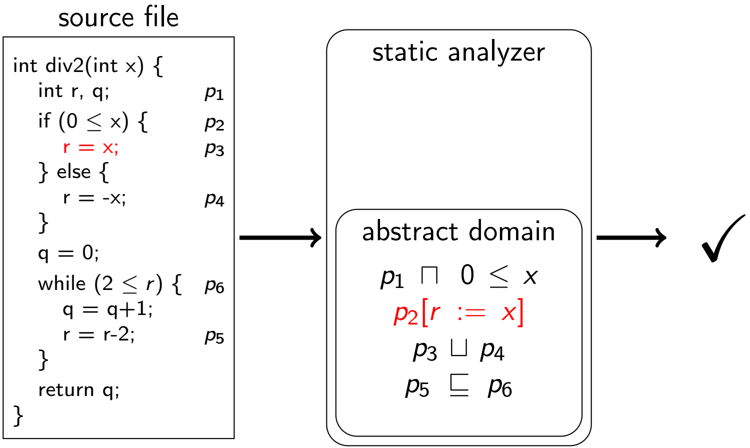
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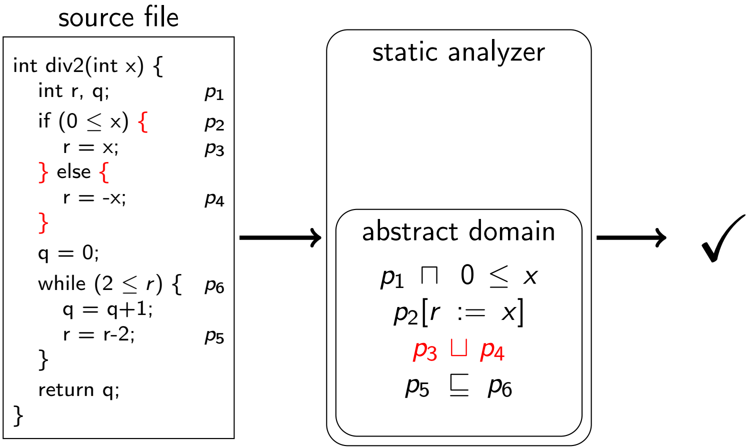
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static analyzer

abstract domain

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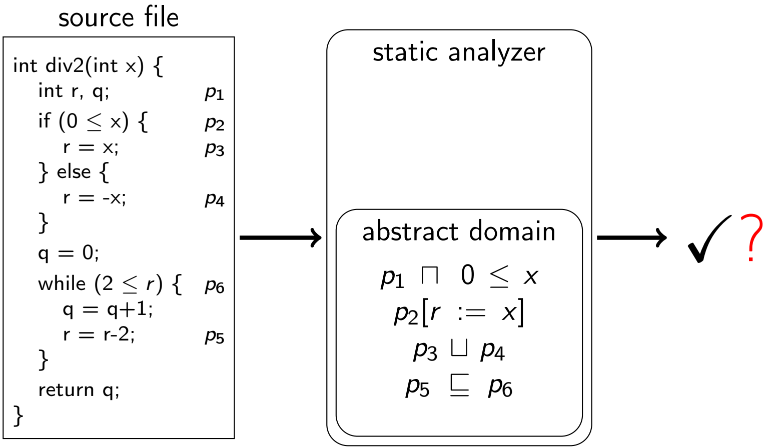
Modular and lightweight certification of polyhedral abstract domains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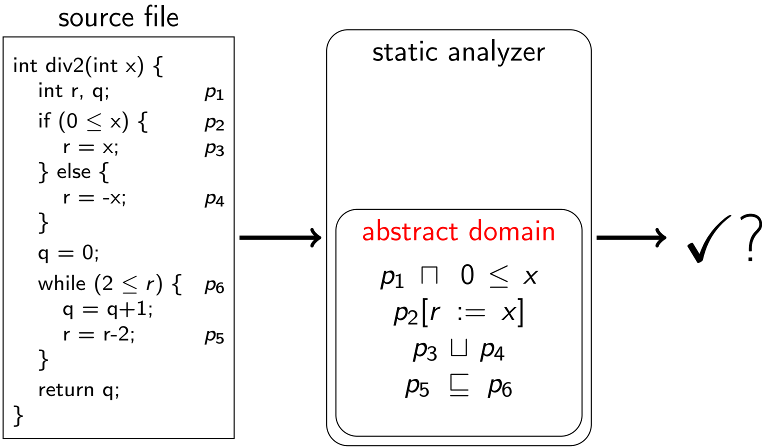
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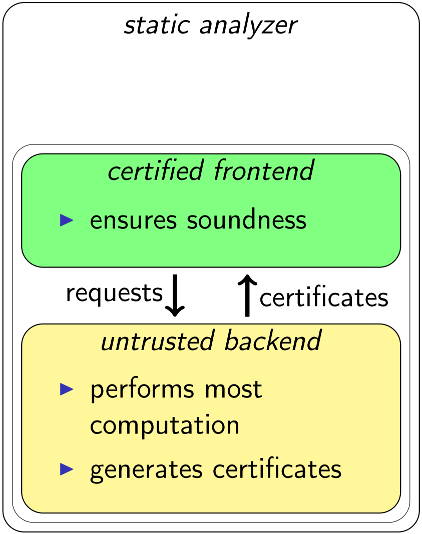
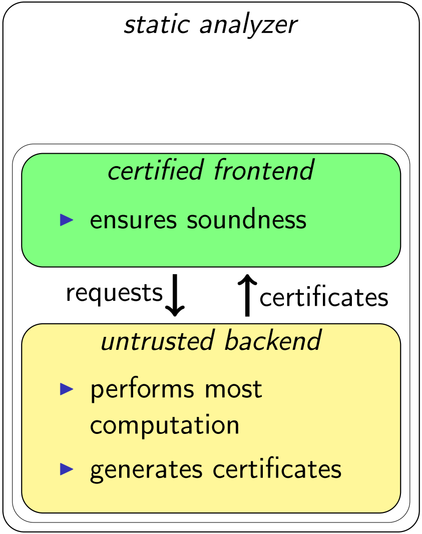


- perfect fit for result verification
- build results from certificates
- formalize impure external code



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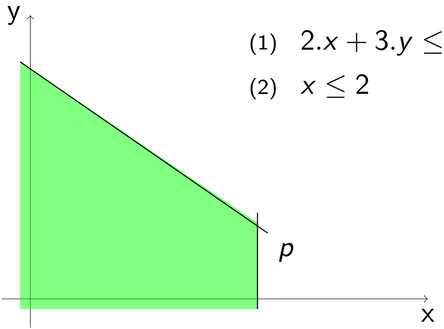
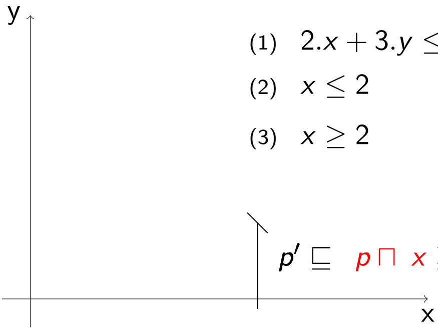


$$\left.\begin{array}{ll}(1) & 2.x + 3.y \leq 6 \\ (2) & x \leq 2\end{array}\right\} p$$



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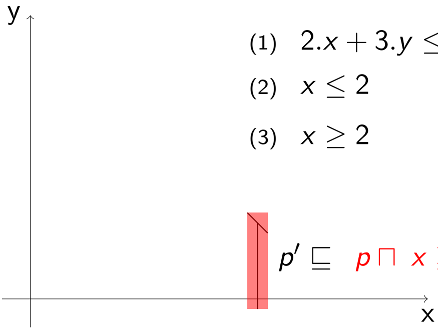
$$(3) \quad x \geq 2$$

$$p' \sqsubseteq \ p \sqcap \ x \geq 2$$



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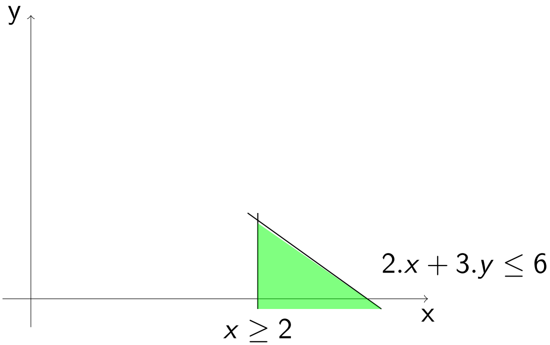
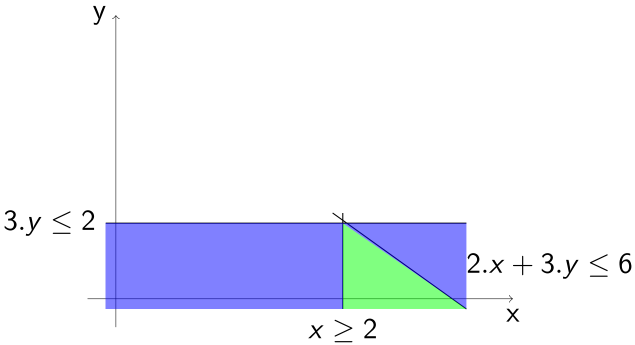
Farkas's lemma:



Farkas's lemma:

$\lambda_1.(2.x + 3.y \leq 6) + \lambda_2.(-x \leq -2) = 3.y \leq 2$



Farkas's lemma:

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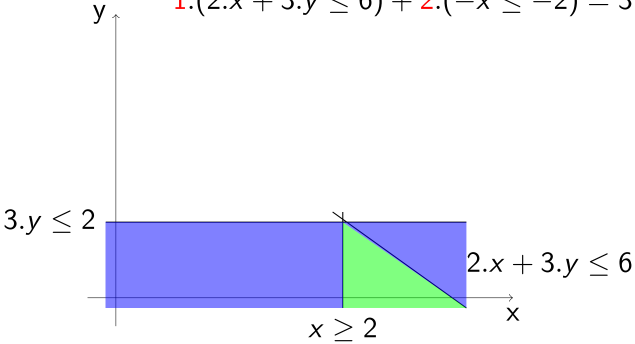
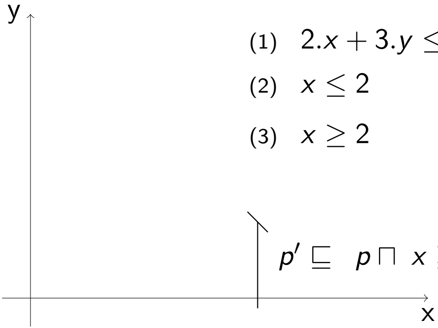
$$1.(2.x + 3.y \leq 6) + 2.(-x \leq -2) = 3.y \leq 2$$





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$$1.(2), 1.(3) \quad x = 2$$

$$1.(1) + 2.(3) \quad 3.y \leq 2$$

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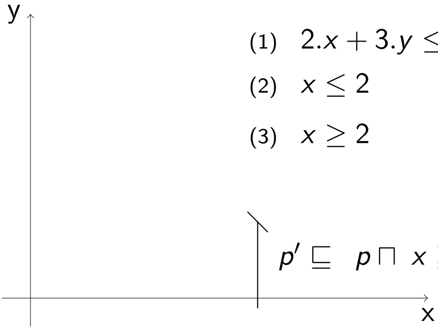

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$1.(2), 1.(3) \quad x = 2$

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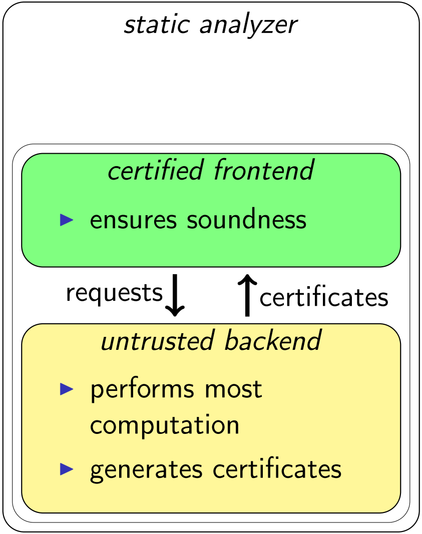
$1.(1) \quad 2.x + 3.y \leq 6$

$1.(2) \quad x \leq 2$

$1.(3) \quad x \geq 2$



- perfect fit for result verification
- build results from certificates
- formalize impure external code

Backend.mli

```
val f : nat → nat;;
```

```
Axiom f : nat → nat.
Extract Constant f ⇒ "Backend.f".
```

```
Goal ∀ n a b, f n = a → f n = b → a = b.
intros. subst. reflexivity. Qed.
```

Backend.mli

```
val f : nat → nat;;
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```
Axiom f : nat → nat.
Extract Constant f ⇒ "Backend.f".

Goal ∀ n a b, f n = a → f n = b → a = b.
intros. subst. reflexivity. Qed.
```

Backend.ml

```
let c = ref 0;;

let f n = begin
   c := n + !c;
   !c;
   end;;
```

Our backend uses GMP.

Backend.mli

```
val f : nat → nat;;
```

```
Axiom f : nat → ?nat.
Extract Constant f ⇒ "Backend.f".

Goal ∀ n a b, f n ⇝ a → f n ⇝ b → a = b.
(* can't prove it *)
```

Backend.ml

```
let c = ref 0;;

let f n = begin
    c := n + !c;
    !c;
    end;;
```

Our backend uses GMP.

may-return monad

$$
\begin{aligned}
\mathtt{?} &: \mathtt{Type} \to \mathtt{Type} \\
\mathtt{unit} &: \mathtt{A} \to \mathtt{?A} \\
\mathtt{bind} &: \mathtt{?A} \to (\mathtt{A} \to \mathtt{?B}) \to \mathtt{?B} \\
\rightsquigarrow &: \mathtt{?A} \to \mathtt{A} \to \mathtt{Prop}
\end{aligned}
$$

$\mathtt{unit}\ \mathtt{a}_1 \rightsquigarrow \mathtt{a}_2 \Rightarrow \mathtt{a}_1 = \mathtt{a}_2$

$\mathtt{bind}\ \mathtt{k}_1\ \mathtt{k}_2 \rightsquigarrow \mathtt{b} \Rightarrow \exists\ \mathtt{a},\ \mathtt{k}_1 \rightsquigarrow \mathtt{a} \wedge \mathtt{k}_2\ \mathtt{a} \rightsquigarrow \mathtt{b}$

may-return monad

$$
\begin{array}{lcl}
\texttt{?} & : & \texttt{Type} \to \texttt{Type} \\
\texttt{unit} & : & \texttt{A} \to \texttt{?A} \\
\texttt{bind} & : & \texttt{?A} \to (\texttt{A} \to \texttt{?B}) \to \texttt{?B} \\
\leadsto & : & \texttt{?A} \to \texttt{A} \to \texttt{Prop}
\end{array}
$$

$$\texttt{unit}\ \texttt{a}_1 \leadsto \texttt{a}_2 \Rightarrow \texttt{a}_1 = \texttt{a}_2$$

$$\texttt{bind}\ \texttt{k}_1\ \texttt{k}_2 \leadsto \texttt{b} \Rightarrow \exists\ \texttt{a},\ \texttt{k}_1 \leadsto \texttt{a} \wedge \texttt{k}_2\ \texttt{a} \leadsto \texttt{b}$$

one implementation: the state monad

$$
\begin{array}{lcl}
\texttt{?A} & = & \texttt{S} \to \texttt{A} \times \texttt{S} \\
\texttt{unit a} & = & \lambda\ \texttt{s.(a,s)} \\
\texttt{bind}\ \texttt{k}_1\ \texttt{k}_2 & = & \lambda\ \texttt{s}_0,\ \texttt{let (a, s}_1\texttt{) = k}_1\ \texttt{s}_0\ \texttt{in k}_2\ \texttt{a s}_1 \\
\texttt{k} \leadsto \texttt{a} & = & \exists\ \texttt{s, fst (k s) = a}
\end{array}
$$

may-return monad

$$
\begin{array}{lcl}
\texttt{?} & : & \texttt{Type} \rightarrow \texttt{Type} \\
\texttt{unit} & : & \texttt{A} \rightarrow \texttt{?A} \\
\texttt{bind} & : & \texttt{?A} \rightarrow (\texttt{A} \rightarrow \texttt{?B}) \rightarrow \texttt{?B} \\
\leadsto & : & \texttt{?A} \rightarrow \texttt{A} \rightarrow \texttt{Prop}
\end{array}
$$

$$\texttt{unit } \texttt{a}_1 \leadsto \texttt{a}_2 \Rightarrow \texttt{a}_1 = \texttt{a}_2$$

$$\texttt{bind } \texttt{k}_1\ \texttt{k}_2 \leadsto \texttt{b} \Rightarrow \exists\ \texttt{a},\ \texttt{k}_1 \leadsto \texttt{a} \land \texttt{k}_2\ \texttt{a} \leadsto \texttt{b}$$

for extracting: the identity monad

$$
\begin{array}{lcl}
\texttt{?A} & = & \texttt{A} \\
\texttt{unit a} & = & \texttt{a} \\
\texttt{bind } \texttt{k}_1\ \texttt{k}_2 & = & \texttt{k}_2\ \texttt{k}_1 \\
\texttt{k} \leadsto \texttt{a} & = & \texttt{k} = \texttt{a}
\end{array}
$$

\+ inlining

Perfect fit for result verification

- simple maths: easy COQ proofs
- build results from certificates: efficient communication
- complex result search

Formalization of external code

- COQ checks we don't use the purity assumption
- may-return monad
  - no runtime overhead
  - low proof overhead

Engineering: a certified abstract domain

- simple/modular formalization
- generic w.r.t. the backend
- experiments show reasonable performance
- integration in the VERASCO analyzer