Church-Rosser Theorem for sequent lambda calculi*

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The Curry-Howard correspondence is a well-known relation between typed lambda calculi and intuitionistic natural deduction. In the last decade, several term calculi have been designed to embody the Curry-Howard correspondence for intuitionistic sequent calculus, [5, 2, 3, 4] among others.

Our focus is on the untyped version of the so called λ^{Gtz}-calculus of [3], which is known to be non-confluent. We regain confluence in the untyped λ^{Gtz}-calculus by imposing restrictions on the reduction rules, which eliminate the critical pair. In this way two subcalculi, the call-by-name λ_N^{Gtz} and the call-by-value λ_V^{Gtz} variant of λ^{Gtz} are obtained. In the $\bar{\lambda}\mu\tilde{\mu}$-calculus of Curien and Herbelin [1], the call-by-name and call-by-value fragments are obtained by dual restrictions on the reductions rule, here instead the situation is different in the call-by-name case. The set of λ_V^{Gtz}-terms is equal to the set of λ^{Gtz}-terms and the the call-by-value λ_V^{Gtz} subcalculus is obtained by restrictions on the reductions. However, the set of λ_N^{Gtz}-terms is a strict subset of the set of λ^{Gtz}-terms. Therefore, we introduce an appropriate mapping to translate λ-terms into λ_N^{Gtz}-terms. We prove then that this mapping preserves the operational semantics of lambda calculus as well as the normal forms.

We prove the confluence of the two proposed subcalculi by adapting Takahashi's parallel reductions technique, used in [6] for proving the confluence of λ-calculus. This is a refinement of the standard Tait and Martin-Löf's proof of the confluence of $\beta\eta$-reduction in the λ-calculus. We analyse the granularity of reduction rules and then define a new notion of parallel reductions in this framework. We then prove the diamond property, which yields the proof of confluence for type free λ_V^{Gtz}-calculus. Finally, we show that the diamond property of the new parallel reduction is also applicable to λ_N^{Gtz}. Confluence of a sequent lambda calculus is usually proved by embedding it in a calculus already known to be confluent. We developed a direct proof of confluence of two subcalculi of λ^{Gtz}, which was our motivation. To the best of our knowledge this is a first direct proof of confluence in sequent lambda calculi.

References

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